Class 2: Power functions and Sketching polynomials
Math 102 Section 107

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September 8, 2017

Announcements

- ▶ (reminder) Course website: https://wiki.math.ubc.ca
- ▶ First WeBWork due Monday
- ▶ Diagnostic Test
- ▶ Office Hours Today: 9-10am, LSK300B

Today...

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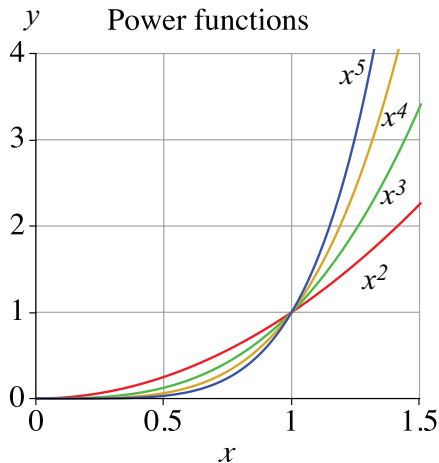- **Even** and **odd** functions.
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- The more "meta" idea: **functions** can be thought of as objects themselves!
- Also - **testing our clickers!**

Last time: **asymptotic** behavior

▸ Small degrees dominate close to $x = 0$; large degrees dominate as $x \to \infty$.

Power functions

Even and odd functions
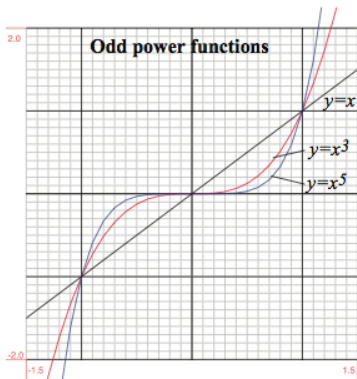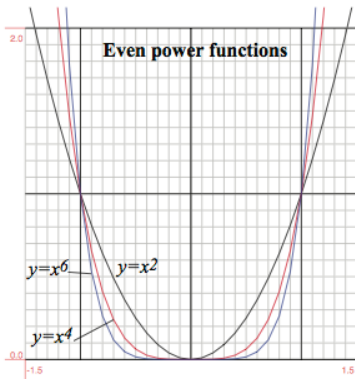
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- An **odd function** $f(x)$ is symmetric about the **origin**:
$$f(x) = -f(-x).$$

Even and odd functions

Q1. The function $f(x) = x^2 + 2x^4$ is

A. an odd function

B. an even function

C. both even and odd

D. neither even nor odd

E. not enough information to tell

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Q3. The function $g(x) = \frac{x^3}{1+x^3}$ is

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- Key idea:
  - Lower powers dominate near $x = 0$.
  - Higher powers dominate for x far from 0.
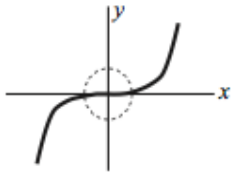
Power functions and curve sketching

Example
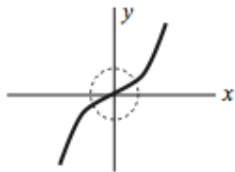
- $y = x^3 + ax$ is in pre-lecture video and the course notes:

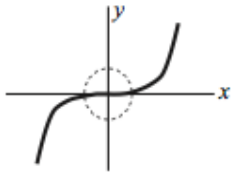$a<0$ $a=0$ $a>0$

Power functions and curve sketching

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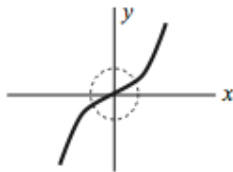- $y = x^3 + ax$ is in pre-lecture video and the course notes:

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- Q4: Sketch a graph of the polynomial $y = x^3 + ax^2$ for $a > 0$ and for $a < 0$. Find all zeroes.

Power functions and curve sketching

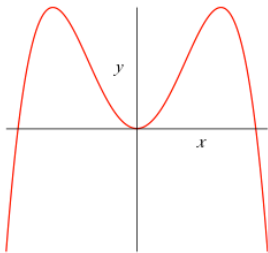Q5. Which of the functions below has this graph?

A. $x^3 - x^5$

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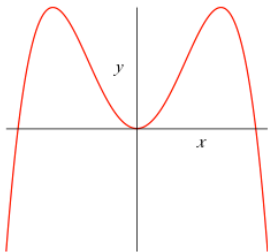Q5. Which of the functions below has this graph?

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- ► Check the last slides for related exam problems.

Related Exam Questions

1. When $x = 1000$, the function
   $g(x) = \frac{6x^4 + 12x^2 + 64x - 87}{2x^3 - 6x^2 + x}$ is closet to

   A. 0.003
   B. 3000
   C. 1000000
   D. 6
   E. 3

2. Sketch the graph of $f(x) = 8x^2 - x^5$.